**Applied Mechanics - Newton's Law**

**Newton's Laws of Motion**

**First Law**: A body at rest or in uniform motion will remain at rest or in uniform motion unless some external force is applied to it.

**Second Law of Motion**: When a body is acted upon by a constant force, its resulting acceleration is proportional to the force and inversely proportional to the mass,

\[ a = \frac{F}{m} \]

Where,

- \( a \) = acceleration, m/s\(^2\)
- \( F \) = force, N
- \( m \) = mass of a body, kg

**Third Law of Motion**: It states that to every action force there is an equal and opposite reaction force.

Motion:

**Displacement**

\[ S = V_0 t + \frac{1}{2} a t^2 \]

**Velocity**

\[ V = V_0 + a t \]

**Acceleration**

\[ a = \frac{dv}{dt} = \frac{d^2 S}{dt^2} \]

Where,

- \( S \) = distance covered by a moving body in time \( t \), m
- \( V \) = Velocity of a moving body, m/s
- \( a \) = acceleration of a moving body, m/s\(^2\)
- \( V_0 \) = initial velocity of a moving body, m/s
- \( t \) = time of movement, s
**Newton’s Law of Gravitation**

Any two bodies attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them.

\[ F \propto \frac{m_1 m_2}{d^2} \]

or

\[ F = G \frac{m_1 m_2}{d^2} \]

where,

- \(F\) = force of attraction, N
- \(m_1\) = mass of body one, kg
- \(m_2\) = mass of body second, kg
- \(d\) = distance between two bodies, m
- \(G\) = Newtonian constant of gravitation

\[ G = 6.66 \times 10^{-11} \, \frac{m^3}{kg \cdot s^2} \]

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**Inertia**

The inertia of a body may be defined as that property of a body which tends to resist a change in its state of rest or motion.

Mass is defined as a quantitative measure of inertia.

**Moment of Inertia of Areas**

\[ I_x = \int y^2 \, dA \]
\[ I_y = \int x^2 \, dA \]
\[ I_z = \int r^2 \, d\iota = I_x + I_y \]

where,

- \(I_x\) = moment of inertia of cross-sectional area about X-axis
- \(I_y\) = moment of inertia of cross-sectional area about Y-axis
- \(I_z\) = moment of inertia of cross-sectional area about Z-axis.
Moment of inertia of areas

Mass moment of inertia \( (I_m) \) of a body is given by:

\[
I_m = \int r^2 \, dm
\]

**Mass moment of inertia for different shapes of body**

**Rectangle**

\[
\begin{align*}
F_x &= \frac{bh^3}{12} \\
L_x &= \frac{bh^3}{3} \\
\bar{y} &= \frac{bh}{12} \left( b^2 + h^2 \right)
\end{align*}
\]

**Triangle**

\[
\begin{align*}
\bar{x} &= \frac{bh^3}{35} \\
L_x &= \frac{bh^3}{12} \\
L_y &= \frac{bh^3}{4}
\end{align*}
\]

**Circle**

\[
\begin{align*}
\bar{x} &= \bar{y} = \frac{\pi r^4}{4} \\
\bar{y} &= \frac{\pi r^4}{2}
\end{align*}
\]

**Ellipse**

\[
\begin{align*}
\bar{x} &= \frac{\pi ab^3}{4} \\
\bar{y} &= \frac{\pi a^3b}{4} \\
\bar{y} &= \frac{\pi ab}{4} \left( a^2 + b^2 \right)
\end{align*}
\]
Circular Cylindrical Shell

\[ I_L = mr^2 \quad \text{where } m = \text{mass}, \ r = \text{radius} \]

Right Circular Cylinder

\[ I_L = \frac{1}{2} mr^2 \]
\[ I_x = \frac{1}{12} m \left( 3r^2 + 4 \frac{l^2}{4} \right) \]

where \( m = \text{mass}, \ r = \text{radius} \)

Semi cylinder

\[ I_L = \left( \frac{1}{2} \times 2mr^2 \right) \]
\[ = \frac{1}{2} mr^2 \]

where \( m = \text{mass}, \ r = \text{radius} \)

Hemisphere

\[ I_x = I_L = \frac{1}{2} \left( \frac{2}{5} \times 2mr^2 \right) \]
\[ = \frac{2}{5} mr^2 \]

where \( m = \text{mass}, \ r = \text{radius} \)

Rectangular Parallelepiped

\[ I_L = \frac{1}{12} m \left( a^2 + b^2 \right) \]
\[ I_x = \frac{1}{12} m \left( 4l^2 + a^2 \right) \]

Uniform Slender Rod

\[ I_{\text{center}} = \frac{ml^2}{12} \]
Right Circular Cone

\[ I_z = \frac{3}{10} mr^3 \]

\[ I_x = I_y = \frac{3}{5} m \left( \frac{r^2}{4} + h^2 \right) \]

Elliptical Cylinder

\[ I_z = \frac{1}{4} m \left( a^2 + b^2 \right) \]

Hemispherical Shell

\[ I_x = I_z = \frac{2}{3} mr^2 \]

Torus (complete)

\[ I_z = m \left( R^2 + \frac{3}{4} a^2 \right) \]

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**Applied Mechanics – Density**

**Density**

\[ \rho = \frac{M}{V} \]

\[ \rho_w = \frac{W}{V} \]

Where,

\[ \rho = \text{density, g/cm}^3 \]

\[ \rho_w = \text{weight density, N/cm}^3 \]

\[ M = \text{mass, g} \]

\[ V = \text{volume, cm}^3 \]

\[ W = \text{weight, N} \]
**Applied Mechanics – Vibrations**

**Vibrations**

1. **Simple Harmonic Motion**

   \[ T = \frac{1}{n} \]

   where,

   \( T \) = period of a vibration, s

   \( n \) = frequency or vibration per unit time, 1/s

2. **Spring Pendulum**

   \[ T = 2\pi \sqrt{\frac{m}{k}} \]

   where,

   \( T \) = period, s

   \( M \) = mass of pendulum

   \( K \) = spring

3. **Simple Pendulum**

   \[ T = 2\pi \sqrt{\frac{l}{g}} \]

   where,

   \( l \) = length of the pendulum

   \( g \) = acceleration due to gravity
4. **Wavelength**

\[ V = n \lambda \]

where,

- \( V \): total distance traveled in one second
- \( \lambda \): length of one wave
- \( n \): number of waves per second

5. **Speed of sound**

\[ V = V_0 + 0.61 t_c \]

where,

- \( V \): speed of sound at temperature \( t_c \) °C, m/s
- \( V_0 \): speed at 0°C, m/s
- \( t_c \): temperature, °C.

6. **Beat Notes**

\[ N = n_2 - n_1 \]

where,

- \( N \): beat frequency, i.e., number of beats per second
- \( n_1, n_2 \): frequencies of two sources producing the sound, vibrations/s

7. **Doppler Effect**

\[ N_0 = n_0 \left( \frac{V \pm v_o}{V \pm v_s} \right) \]

where,

- \( N_0 \): frequency heard by the observer
- \( n_0 \): frequency of the source
- \( V \): velocity of sound
- \( V_s \): velocity of source
- \( V_o \): velocity of the observer
8. **Intensity of sound**

\[ E = \frac{E_0}{d^2} \text{ (Inverse square law)} \]

where,

- \( E \) = intensity of sound at any distance \( d \), microwatts/cm\(^2\) or decibels
- \( E_0 \) = intensity of sound at unit distance, decibels

9. **Vibrating Strings**

\[ V = n \lambda \]
\[ V = \sqrt{\frac{E}{m}} \]
\[ \lambda = \frac{2L}{n} \]

where,

- \( V \) = velocity of sound, m/s
- \( n \) = frequency or number of waves passing by per second
- \( \lambda \) = length of one wave or wavelength
- \( F \) = tension in a rope or string, N
- \( M \) = mass of string per unit length, kg/m
- \( L \) = distance between two consecutive nodes, m

10. **Sound Wave Through Gas**

\[ V = \sqrt{\frac{K P}{\rho}} \]

where,

- \( V \) = wave velocity, cm/s
- \( P \) = gas pressure, dynes/cm\(^2\)
- \( \rho \) = gas density, g/cm\(^3\)
- \( K \) = proportionality constant